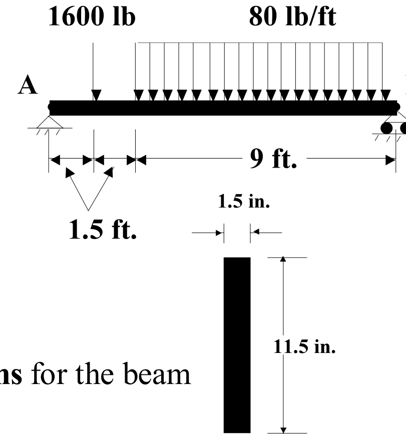
Homework No. 1 MAE/CE 459/559 John A. Gilbert, Ph.D.

Fall 2004

1. A beam is loaded as shown. The dimensions of the cross section appear in the insert.



   (a) Draw a **complete free body diagram** showing an equivalent system constructed by replacing the distributed load, and determine the **reactions** at the pin (A) and roller (B).

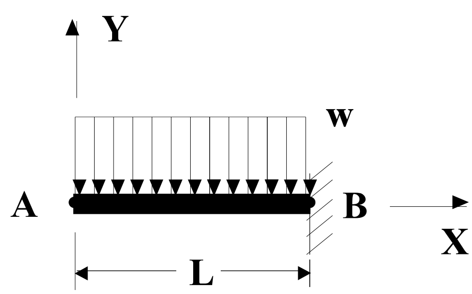
   (b) Establish the **analytical relationships** for the shear force and moment distribution for all points along the span of the beam illustrated in the figure.

   (c) Draw the **shear and bending-moment diagrams** for the beam illustrated in the figure.

   (d) Determine the **maximum transverse shearing stress** in the beam assuming that it has the cross section shown.

   (e) Determine the **maximum normal stress** due to bending.

2. A cantilever beam that is fixed into a wall at B is subjected to uniformly distributed load as shown. The weight of the beam can be neglected. The origin of the coordinate system is at point A.



   (a) Draw a **complete free body diagram** of the beam and determine the **reactions** at the fixed support.

   (b) Determine the **equation of the elastic curve** (in terms of L,E,I,w).

   (c) Determine the **deflection at the free end** (in terms of L,E,I,w).

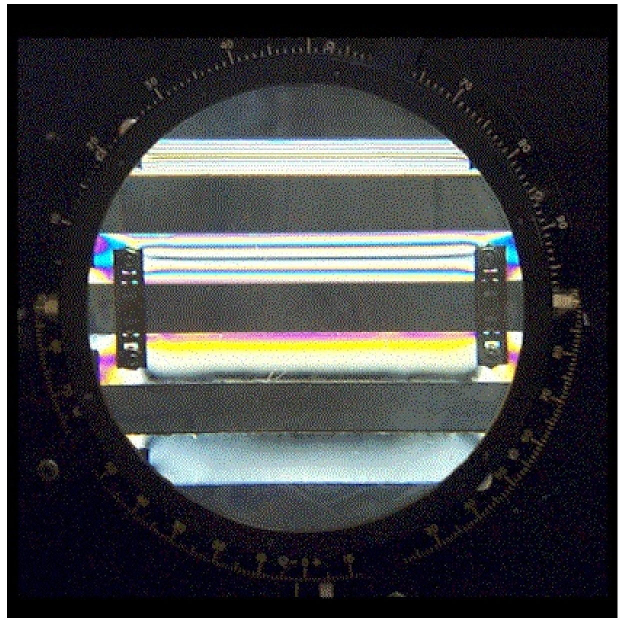
Answers:

Problem 1: (a) $\underline{R}_{Ax}$ = 0, $\underline{R}_{Ay}$ = 1670 lb ↑, $\underline{R}_{B}$ = 650 lb ↑; (b) in the first segment between A and the 1600 lb load, for example, V = 1670 lb (↓) and M = 1670 x ft·lb (cclw), etc.; (c) plot the relationships found in part (c) along the span; (d) τ_{max} = 145.2 psi; (e) σ_{max} = 959 psi.

Problem 2: (a) $\underline{R}_{Bx}$ = 0, $\underline{R}_{By}$ = wL ↑, $\underline{M}_{B}$ = $wL^2/2$ clw ; (b) y = - w/24EI [x^4-$4L^3x$+$3L^4$] ; (c) y_A = - $wL^4/8EI$.

Homework No. 2 MAE 459/559 John A. Gilbert, Ph.D.

Fall 2004



Photoelastic fringes on beams.

1. The figure to the right shows the isochromatic fringe patterns in four beams made from a plastic called PSM-1 that has a material fringe value of 40 psi/fringe/in. The elastic modulus of the material is 360 ksi and it has a Poisson's ratio of 0.38. All of the beams shown in the figure have a constant thickness of 0.375 in., and each beam is subjected to a moment of 13.33 in.-lb. A portion of the loading fixture can be seen at the edges of the field of view.

The distribution and number of fringes in the beams are directly proportional to the stress. The heavy black fringe in each beam represents the neutral axis which passes through the centroid of the section. The stress varies linearly with depth; compression on one side of the beam, tension on the other.

The beam situated second from the top is a 1 in. deep control standard and represents an unreinforced, homogeneous beam made of concrete having an elastic modulus equal to that of PSM-1. A digital compensator revealed a fringe order number of 2.04 at the boundaries. The top beam is only 0.75 in. deep and illustrates that the maximum stress becomes much higher when the depth is smaller. This beam has the least moment of inertia of them all and, consequently, the highest density of fringes. Compensation revealed a fringe order number of 3.75 at the boundaries.

A 0.025 in. thick, 0.375 in. wide, steel strip is bonded to the lower surface of the beam situated third from the top. The steel has an elastic modulus of 30 Msi, making the elastic modula ratio, $n = 83.3$. The centroid shifts toward the steel, and the neutral axis is observed to be further down in the composite section. Compensation revealed a fringe order number of 1.02 at the upper boundary.

The bottom beam, on the other hand, has steel strips bonded on both the top and bottom faces. The isochromatic fringe pattern in the plastic is barely visible. The neutral axis of the beam should be directly in the center of the beam, but it is observed to be shifted slightly toward one side. Compensation revealed that the fringe orders are 0.09 and 0.17 at the upper and lower surfaces, respectively. For purposes of comparison, it can be assumed that the fringe order is 0.13.

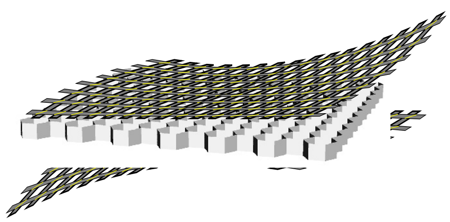
Compute the theoretical maximum stress in the concrete for each beam using the mechanics of materials approach for composite sections. Use the governing equation for photoelasticity and the isochromatic fringe patterns to compute the corresponding experimentally measured values. Compare these results and specify the percentage error based on the theoretical value.

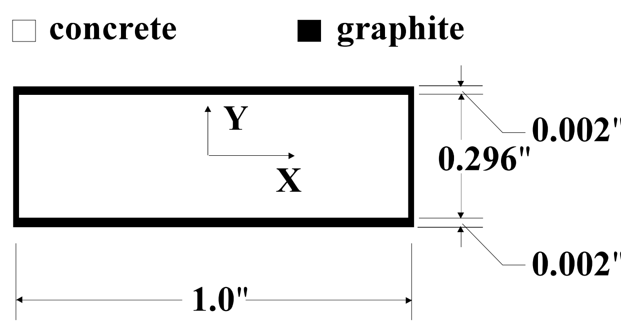
Answers: No. 1: $\sigma_{theroy} = 379$, $\sigma_{exp} = 400$ psi, 6%; No. 2: $\sigma_{theroy} = 213$ psi, $\sigma_{exp} = 218$ psi, 2%; No. 3: $\sigma_{theroy} = 115$ psi, $\sigma_{exp} = 109$ psi, 5%; No. 4: $\sigma_{theroy} = 15$ psi, $\sigma_{exp} = 14$ psi, 7%.

2. The figures show a pictorial representation, and a first approximation model, of the configuration being used to construct of our 1999 concrete canoe. The design relies a Mylar honeycomb sandwiched between two dry woven graphite fiber mats. Concrete is placed within the honeycomb and over the graphite. It is assumed that pure bending takes place around a horizontal axis and that the applied moment is 23 in-lb. The modulus of elasticity of the concrete is 4.4 x 10^3 psi, whereas the modulus of the graphite mat is 33 x 10^6 psi. Neglect the strength added by the honeycomb.

Hint: The cross section is symmetric and centroid lies at the center. You know n = 7500; and, should compute the centroidal moment of inertia for the transformed section as 0.6608 in^4.

(a) Determine the **maximum compressive stress** in the **concrete** (answer: 5.2 psi).
(b) Determine the **maximum compressive stress** in the **graphite** (answer: 38.7 ksi).

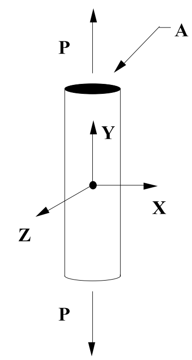




Homework No. 3 **MAE 459/559** **John A. Gilbert, Ph.D.**

Fall 2004

1. The central portion of the tensile specimen shown in the accompanying figure represents a uniaxial stress condition; P is the applied load and A is the cross-sectional area. For a point located on the surface and in the central portion of the specimen:



(a) Draw a two-dimensional stress element having horizontal and vertical faces. Show the stress distribution on this element. Are these the principal stress planes?

(b) Construct the Mohr's circle for the point and draw a rotated element showing the maximum in-plane shear stress. Be sure to label all stresses and specify the angle of orientation with respect to, say, the horizontal direction.

(c) How does the maximum in-plane shear stress compare with the absolute maximum shear stress at the point?

(d) Develop the constitutive equation(s) for this uniaxial stress condition from the three dimensional Hooke's Law:

$$\epsilon_{xx} = \frac{1}{E}[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})]$$

$$\epsilon_{yy} = \frac{1}{E}[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})]$$

$$\epsilon_{zz} = \frac{1}{E}[\sigma_{zz} - \nu(\sigma_{yy} + \sigma_{xx})]$$

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$$

$$\gamma_{yz} = \frac{2(1+\nu)}{E}\tau_{yz}$$

$$\gamma_{zx} = \frac{2(1+\nu)}{E}\tau_{zx}$$

(e) Develop the mathematical relationships for Young's Modulus and Poisson's Ratio and describe how these quantities could be calculated by making physical measurements.

(f) Specify the six components of strain if the specimen is made of steel ($E = 30 \times 10^6$ psi, $\nu = 0.3$) and loaded to yield ($\sigma_y = 45 \times 10^3$ psi).

(g) What would be the angular orientation of the fracture surface with respect to a horizontal axis if the material were ductile and failed along the planes of maximum shear? What would happen if the material were brittle?

2. Consider a cantilever specimen of length, L, subjected to a concentrated force, P, at the free end.

   (a) Draw a free body diagram and determine the reactions at the fixed support.
   (b) Draw the shear force and bending moment diagrams.
   (c) Determine the elastic curve.
   (d) Specify the deflection at the free end in terms of x, L, E, I, and P.

Selected answers:

Problem 1: (a) $\sigma_{xx} = 0$, $\sigma_{yy} = P/A$, $\tau_{xy} = 0$, yes, (b) element oriented at 45° with $\tau = \sigma = P/2A$, (c) equal, (d) $\sigma_{yy} = E\epsilon_{yy}$ ..., (e) $E = \sigma_{yy}/\epsilon_{yy}$, $\upsilon = -\epsilon_{xx}/\epsilon_{yy} = -\epsilon_{zz}/\epsilon_{yy}$, measure strain and compute stress, (f) $\epsilon_{xx} = \epsilon_{zz} = -450\ \mu\epsilon$, $\epsilon_{yy} = 1500\ \mu\epsilon$, $\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0\ \mu\epsilon$, (g) ductile @ 45°, brittle @ 0°.

Problem 2: (a) $R_x = 0$; $R_y = P$; $M = PL$ (b) $V = P$; $M = Px - PL$ (c) $y = Px^2/6EI\ (x - 3L)$ (d) $y = -PL^3/3EI$.